Chapter 5

5.1 Use the Fourier transform to calculate the Fourier transform of:

(a) $\left(\frac{1}{2}\right)^{n-1} u[n-1]$ (b) $\left(\frac{1}{2}\right)^{|n-1|}$

5.3 Determine the Fourier transform for $-\pi \le \omega \le \pi$ in the case of each of the following periodic signals

(a) $\sin\left(\frac{\pi}{3}n+\frac{\pi}{4}\right)$ (b) $2+\cos\left(\frac{\pi}{6}n+\frac{\pi}{8}\right)$

5.6 Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transform of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties.

(a) $x_1[n] = x[1-n] + x[-1-n]$

(b) $x_2[n] = \frac{x^*[-n] + x[n]}{2}$

(c) $x_3[n] = (n-1)^2 x[n]$

5.14 Suppose we are given the following facts about an LTI system S with impulse response $h[n]$ and frequency response $H(e^{j\omega})$:

1. $\left(\frac{1}{4}\right)^n u[n] --> g[n]$, where $g[n] = 0$ for $n \ge 2$ and $n \le 0$.
2. $H(e^{j\pi/2}) = 1$.
3. $H(e^{j\omega}) = H(e^{j(\omega-\pi)})$

Determine $h[n]$.

5.16 The Fourier transform of a particular signal is $X(e^{j\omega}) = \sum_{k=0}^{3} \frac{(1/2)^k}{1-\frac{1}{4}e^{-j(\omega-\pi/2k)}}$. It can be shown that $x[n] = g[n]q[n]$, where $g[n]$ is of the form $\alpha^n u[n]$ and $q[n]$ is a periodic signal with period N.

(a) Determine the value of α.

(b)Determine the value of N.

(c)Is $x[n]$ real?

5.21 Compute the Fourier transform of each of the following signals:

(b) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

(f) $x[n] = \begin{cases} n, -3 \le n \le 3 \\ 0, otherwise \end{cases}$

(i) $x[n] = x[n-6]$, and $x[n] = u[n] - u[n-5]$ for $0 \le n \le 5$

5.22 The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

(a) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, \frac{3\pi}{4} \le |\omega| \le \pi, 0 \le |\omega| \le \frac{\pi}{4} \end{cases}$

(b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

(e) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi}{2}k)$

5.23 Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure 5.23. Perform the following calculation without explicitly evaluating $X(e^{j\omega})$.

(a) Evaluate $X(e^{j0})$;

(b) Find $\measuredangle X(e^{j\omega})$

(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$

(d) Find $X(e^{j\pi})$

(e) Determine and sketch the signal whose Fourier transform is $\mathrm{Re}\{X(e^{j\omega})\}$.

(f) Evaluate $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega$ and $\int_{-\pi}^{\pi} |\frac{dX(e^{j\omega})}{d\omega}|^2 \, d\omega$

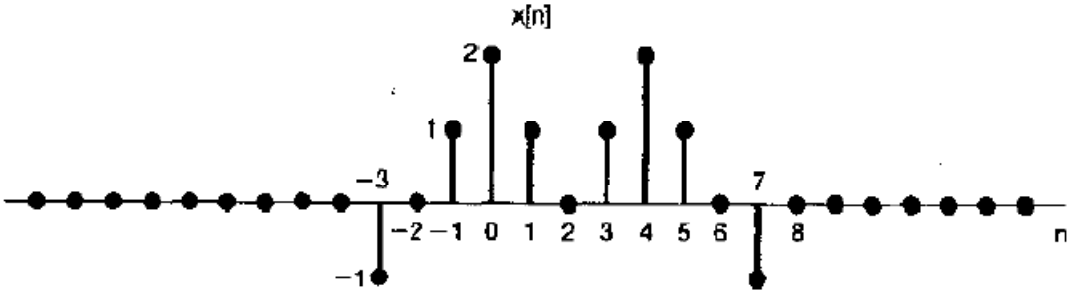

Fig. 5.23

5.29 Consider a discrete-time LTI system with impulse response $h[n]=\left(\frac{1}{2}\right)^n u[n]$. Use Fourier transforms to determine the response to each of the following input signals:

(i) $x[n]=\left(\frac{3}{4}\right)^n u[n]$ (ii) $x[n]=(n+1)\left(\frac{1}{4}\right)^n u[n]$ (iii) $x[n]=(-1)^n$

5.34 Consider a system consisting of the cascade of two LTI systems with frequency responses $H_1(e^{j\omega}) = \dfrac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$ and $H_2(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-j2\omega}}$.

(a) Find the difference equation describing the overall system.

(b) Determine the impulse response of the overall system.