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Chapter 4

1. Use the Fourier transform analysis equation to calculate the Fourier transforms of (a) $e^{-2(t-1)}u(t-1)$ (b) $e^{-2|t-1|}$

2. Use the Fourier transform analysis to calculate the Fourier transforms of (a) $\delta(t+1)+\delta(t-1)$ (b) $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

3. Use the Fourier transform analysis to determine the inverse Fourier transform of: (a) $X_1(j\omega)=2\pi\delta(\omega)+\pi\delta(\omega-4\pi)+\pi\delta(\omega+4\pi)$ (b)

$$X_2(j\omega)=\begin{cases} 2, & 0\le\omega\le 2 \\ -2, & -2\le\omega\le 2 \\ 0, & |\omega|>2 \end{cases}$$

4. Given that x(t) has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties. (a) $x_1(t)=x(1-t)+x(-1-t)$

   (b) $x_2(t)=x(3t-6)$ (c) $x_3(t)=\frac{d^2}{dt^2}x(t-1)$

5. Let x(t) be a signal with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

   1. X(t) is real and nonnegative

2. The inverse Fourier transform of $(1+j\omega)X(j\omega)$ is $Ae^{-2t}u(t)$, where A is independent of t.

3. $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$

Determine a closed-form expression for $x(t)$.

6. Compute the Fourier transform of each of the following signals:

(a) $\sum_{k=0}^{\infty} \alpha^k \delta(t-kT)$, $|\alpha|<1$

(b) $[te^{-2t}\sin(4t)]u(t)$

(c) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

7. Let $X(j\omega)$ denote the Fourier transform of the signal x(t) depicted in Fig. 1.

(a) Find $\angle X(j\omega)$

(b) Find $X(j0)$

(c) Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$

(d) Evaluate $\int_{-\infty}^{\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$

(e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

(f) Sketch the inverse Fourier transform of the real part of $X(j\omega)$.

Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

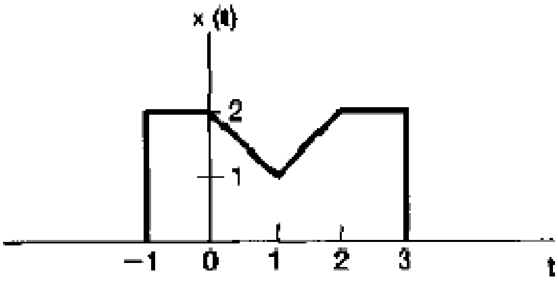

Fig. 1

8. Compute the convolution of each of the following pairs of signals $x(t) = te^{-2t}u(t)$ and $h(t) = e^{-4t}u(t)$ by calculating $X(j\omega)$ and $H(j\omega)$, using the convolution property, and inverse transforming.

9. (a) Let x(t) have the Fourier transform $X(j\omega)$, and let p(t) be periodic with fundamental frequency ω_0 and Fourier series representation $p(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$. Determine an expression for the Fourier transform of $y(t) = x(t)p(t)$.

   (b) Suppose that $X(j\omega)$ is as depicted in Fig. 2. Sketch the spectrum of $y(t) = x(t)p(t)$. for each of the following choices of $p(t)$.

   (I) $p(t) = \cos(t)$ (2) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \pi n)$

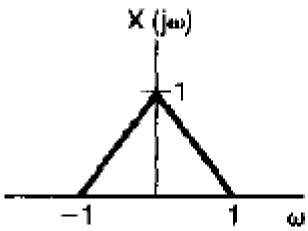

Fig. 2

10.Consider an LTI system S with impulse response $(t-1)\ h(t)=\dfrac{\sin(4(t-1))}{\pi(t-1)}$.

Determine the output of S for each of the following inputs:

(a) $x(t)=\cos(6t+\frac{\pi}{2})$ (b) $x(t)=\left(\dfrac{\sin 2t}{\pi t}\right)^2$

11. A causal and stable LTI system S has the frequency response $H(j\omega)=\dfrac{j\omega+4}{6-\omega^2+5j\omega}$.

(a) Determine a differential equation relating the input x(t) and output y(t) of S.

(b) Determine the impulse response h(t) of S.

(c) What is the output of S when the input is $x(t)=e^{-4t}u(t)-te^{-4t}u(t)$.