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Chapter 3

1. A continuous-time periodic signal x(t) is real valued and has a fundamental period T = 8. The nonzero Fourier series coefficient for x(t) are $a_1 = a_{-1} = 2$, $a_3 = a_{-3} = 4j$. Express x(t) in the form $x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$.

2. A discrete-time period signal x[n] is real valued and has a fundamental period N = 5. The nonzero Fourier series coefficients for x[n] are $a_0 = 1$, $a_2 = a_{-2}^* = e^{j\pi/4}$, $a_4 = a_{-4}^* = 2e^{j\pi/3}$. Express x[n] in the form $x[n] = A_0 + \sum_{k=0}^{\infty} A_k \sin(\omega_k n + \phi_k)$.

3. For the continuous-time periodic signal $x(t) = 2 + \cos(\frac{2\pi}{3}t) + 4\sin(\frac{5\pi}{3}t)$, determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$.

4. Use the Fourier series analysis equation to calculate the coefficients a_k for the continuous-time periodic signal

$$x(t) = \begin{cases} 1.5, & 0 \le t \le 1 \\ -1.5 & 1 \le t \le 2 \end{cases}$$

with fundamental frequency $\omega_0 = \pi$.

5. Suppose the periodic signal x(t) has fundamental period T and Fourier coefficients a_k. In a variety of situations, it is easier to calculate the Fourier series coefficient b_k for $\frac{dx(t)}{dt}$, as opposed to calculating a_k directly. Given that $\int_T^{2T} x(t)dt = 2$, find an expression for a_k in terms of b_k and T using the properties of Fourier series representation.

6. Consider a continuous-time LTI system whose frequency response is $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = \frac{\sin(4\omega)}{\omega}$. If the input to this system is a periodic signal $x(t) = \begin{cases} 1 & 0 \le t \le 4 \\ -1 & 4 \le t \le 8 \end{cases}$ with period T=8, determine the corresponding system output y(t).

7. Consider a continuous-time ideal lowpass filter S whose frequency response is $H(j\omega) = \begin{cases} 1 & |\omega| \le 100 \\ 0 & |\omega| > 100 \end{cases}$. When the input to this filter is a signal x(t) with fundamental period $T = \pi / 6$ and Fourier series coefficient a_k, it is found that $x(t) \xrightarrow{S} y(t) = x(t)$. For what values of k is it guaranteed that $a_k = 0$?

8. Determine the Fourier series for the following signals x(t) illustrated in Fig. 1-3.

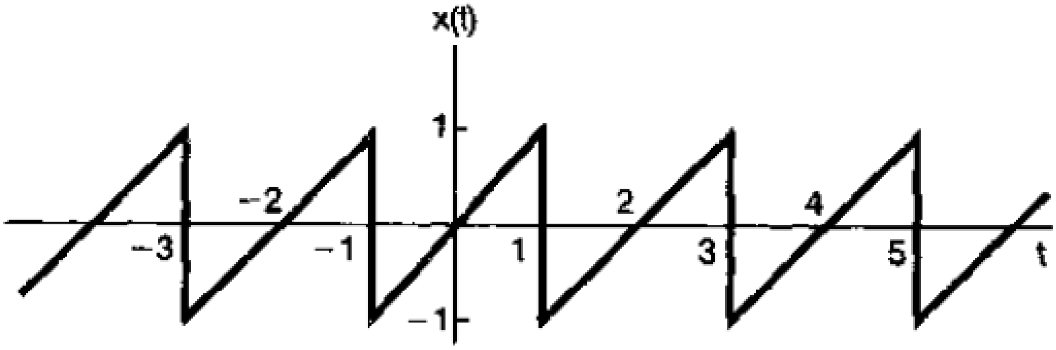

Fig. 1

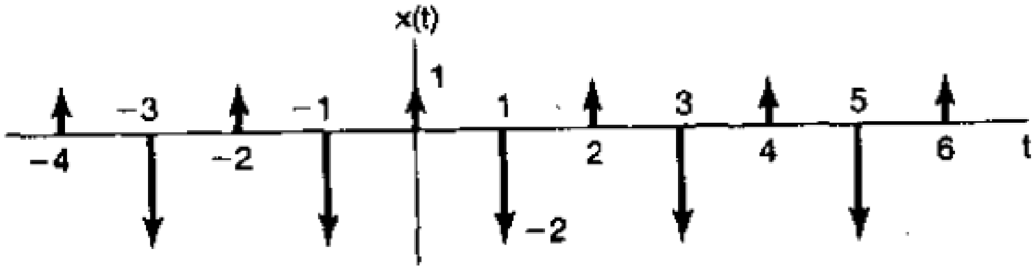

Fig. 2

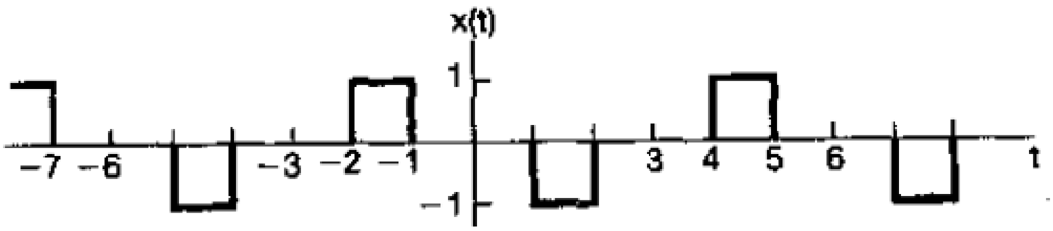

Fig. 3

9. Let $x(t)=\begin{cases} t & 0\le t\le 1 \\ 2-t & 1\le t\le 2 \end{cases}$ be a periodic signal with fundamental period T=2 and Fourier coefficients a_k. (a) Determine the values of a_0. (b) Determine the Fourier series representation of $\frac{dx(t)}{dt}$. (c) Use the result of part (b) and the differentiation property of the

continuous-time Fourier series to help determine the Fourier series coefficients of x(t).

10. Determine the Fourier series coefficients for each of the following discrete-time periodic signals.

(a) X[n] is depicted in Fig. 4

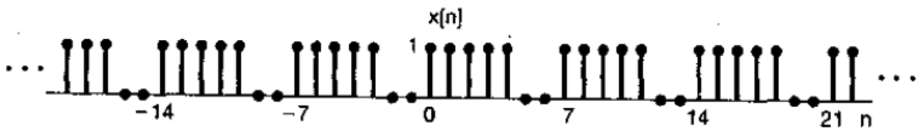

Fig. 4

(b) $x[n] = \sin(2\pi n/3)\cos(\pi n/2)$.

11. Let $x[n] = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & 8 \le n \le 9 \end{cases}$ be a periodic signal with fundamental period $N = 10$ and Fourier series coefficients a_k. Also, let $g[n] = x[n] - x[n-1]$.

(a) Determine the fundamental period of g[n].

(b) Determine the Fourier series coefficients of g[n].

(c) Using the Fourier series coefficients of g[n] and the first-difference property to determine a_k for $k \ne 0$.